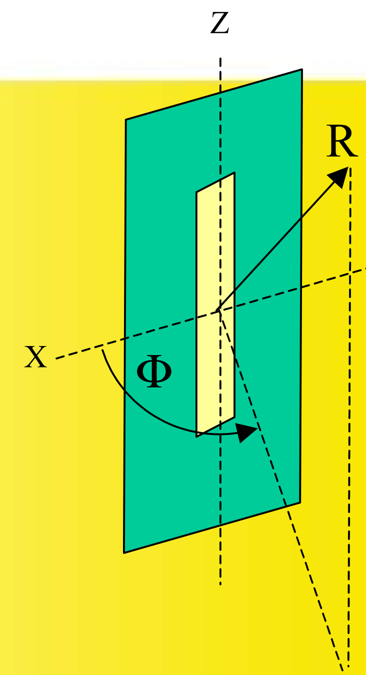
Consider a Vertical Slot Antenna

•**From Ramo, Whinnery and Van Duzer, P 620**



$$\overline{E} = \frac{E_0}{R} e^{j(\omega t - \beta R)} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \overline{a_\Phi}$$

$$\overline{E} = \frac{E_s}{R} e^{-j\beta R} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \overline{a_\Phi}$$

$$\overline{a_\Phi} = -\sin\phi\,\overline{a_x} + \cos\phi\,\overline{a_y} = \frac{-y}{\sqrt{x^2+y^2}}\overline{a_x} + \frac{x}{\sqrt{x^2+y^2}}\overline{a_y}$$

$$R = \sqrt{x^2+y^2+z^2}$$

$$\cos\theta = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\sin\theta = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$$

Where:

$\overline{E}$=Electric field strength vector

R=radial distance from antenna

ω=radian frequency

β=Propagation constant

E_ϕ= Electric field component

Expressing in Rectangular Coordinates

$$\overline{E} = \frac{E_s}{\sqrt{x^2+y^2+z^2}} e^{-j\beta\sqrt{x^2+y^2+z^2}} \frac{\cos\left(\frac{\pi}{2}\frac{z}{\sqrt{x^2+y^2+z^2}}\right)}{\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}} \left(\frac{-y}{\sqrt{x^2+y^2}}\overline{a_x} + \frac{x}{\sqrt{x^2+y^2}}\overline{a_y} \right)$$

$$E_{x=} E_s e^{-j\beta\sqrt{x^2+y^2+z^2}} \frac{\cos\left(\frac{\pi}{2}\frac{z}{\sqrt{x^2+y^2+z^2}}\right)}{x^2+y^2}(-y)$$

$$E_{y=} E_s e^{-j\beta\sqrt{x^2+y^2+z^2}} \frac{\cos\left(\frac{\pi}{2}\frac{z}{\sqrt{x^2+y^2+z^2}}\right)}{x^2+y^2}(x)$$

$$E_z = 0$$

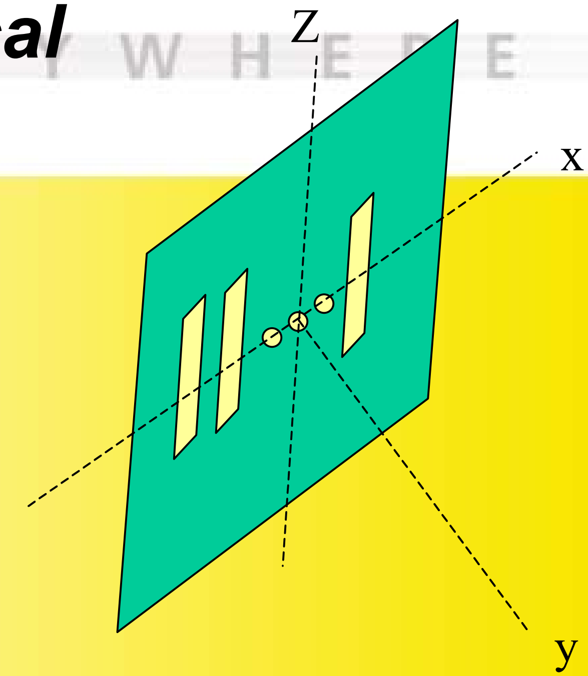
Consider an arbitrarily locaytion (x_n, y_n, z_n)

$$E_x = E_s e^{-j\beta\sqrt{(x-x_n)^2+(y-y_n)^2+(z-z_n)^2}} \frac{\cos\left(\frac{\pi}{2}\frac{z-z_n}{\sqrt{(x-x_n)^2+(y-y_n)^2+(z-z_n)^2}}\right)}{(x-x_n)^2+(y-y_n)^2}(y_n - y)$$

$$E_y = E_s e^{-j\beta\sqrt{(x-x_n)^2+(y-y_n)^2+(z-z_n)^2}} \frac{\cos\left(\frac{\pi}{2}\frac{z-z_n}{\sqrt{(x-x_n)^2+(y-y_n)^2+(z-z_n)^2}}\right)}{(x-x_n)^2+(y-y_n)^2}(x - x_n)$$

$$E_z = 0$$

An Array of Vertical Slot Antennas



Consider an array of vertical slot antennae located in positions (X_n,0, Z_n)

$$E_x = \sum_n E_n e^{-j\beta\sqrt{(x-x_n)^2+(y)^2+(z-z_n)^2}} \frac{\cos\left(\frac{\pi}{2}\frac{z-z_n}{\sqrt{(x-x_n)^2+(y)^2+(z-z_n)^2}}\right)}{(x-x_n)^2+(y)^2}(-y)$$

$$E_y = \sum_n E_n e^{-j\beta\sqrt{(x-x_n)^2+(y)^2+(z-z_n)^2}} \frac{\cos\left(\frac{\pi}{2}\frac{z-z_n}{\sqrt{(x-x_n)^2+(y)^2+(z-z_n)^2}}\right)}{(x-x_n)^2+(y)^2}(x-x_n)$$

$$E_z = 0$$