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TECHNICAL MEMORANDA

Calculation of
"Necessary Bandwidth"
for Pseudo Noise Bi-Phase Modulated (Direct Sequence)
Spread Spectrum Radars

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Introduction

This memoranda addresses the computation of the radiated bandwidth required by a pseudo-noise (PN) bi-phase modulated (direct sequence) precision ranging radar. Of interest is the amount of bandwidth required, or what is "necessary" bandwidth. This note defines "necessary" for the case of precision ranging radars, shows that spectral components lying outside the "necessary bandwidth" do not contribute to system performance, and computes performance degradation for one case where the radiated bandwidth used is less than the necessary bandwidth.

Necessary Bandwidth Criteria

It is well known in general, regardless of the waveform type used, that range resolution in radars is inversely proportional to radiated bandwidth.[1] Range resolution in radars herein refers to the ability of the radar system to distinguish the presence of two targets which are closely spaced in range from the presence of one target at a particular range. The amount of range separation between two (otherwise identical) targets required for the radar to determine the presence of two objects as opposed to one is the quantitative measure of "range resolution". Herein, the performance requirements made of the BUA radar regarding range resolution are used to define the meaning of "necessary bandwidth" in the application. It simply remains to quantify the minimum amount of frequency span required in the radiated transmission to achieve a particular range resolution capability given the nature of the waveform modulation in the BUA radar.

Range Resolution in PN Coded Direct Sequence Radar

Radiated Spectrum

The radar at hand (Back Up Aid radar) uses bi-phase modulation where the modulating waveform is a serial digital bit stream that is pseudo-noise in nature. The example radar studied here uses a PN code of length 8191 bits. The radiated signal power spectral density of this type of modulation is given by [2]

$$G_{BPSK}(f) = P_s T_b \, {\frac{\sin[\pi(f - f_o)T_b]}{\pi(f - f_o)T_b}}^{2} \qquad (1)$$

where

T_b = the PN sequence bit period = 1.47 nsec in the example

f_o = the carrier frequency.

This is shown in figure 1 for the case of a 17 Ghz carrier with a bit modulation rate of 680 Mhz. As indicated by eq. (1), the spacing between the spectral nulls seen in figure 1 is governed by the modulating bit rate which is the inverse of the bit period. The mainlobe null-to-null frequency span is twice the modulation bit rate. Each spectral sidelobe has a null-to-null bandwidth of the modulating bit rate.

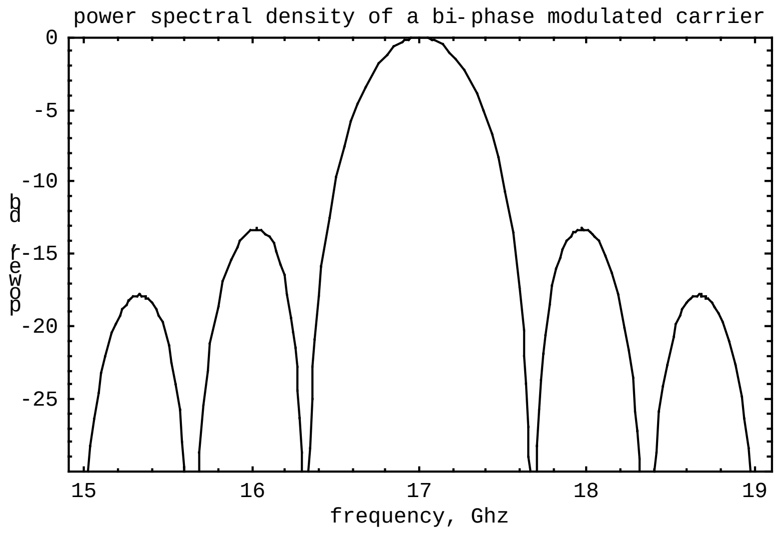

Figure 1. Power spectral density of a bi-phase modulated carrier. The modulation bit stream is random or psuedo-random in nature.

System Response – Cross Correlation Function

The system response to target range is governed by the bit rate of the modulation. Thus there is a direct one to one correspondence between spectral width and system range bin response, as expected. Specifically, the system response vs. target range for a single range bin is shown in figure 2. The example response is for range bin # 16, where internal time delays applied to the correlation receiver code correspond to 16 bit periods in the PN code. Each range bin in the system has a base width of 2 T_b in time, where range bin width here is characterized by the differential propagation delay from a target as it changes range. The response in figure 2 is shown in terms of units of range bins.

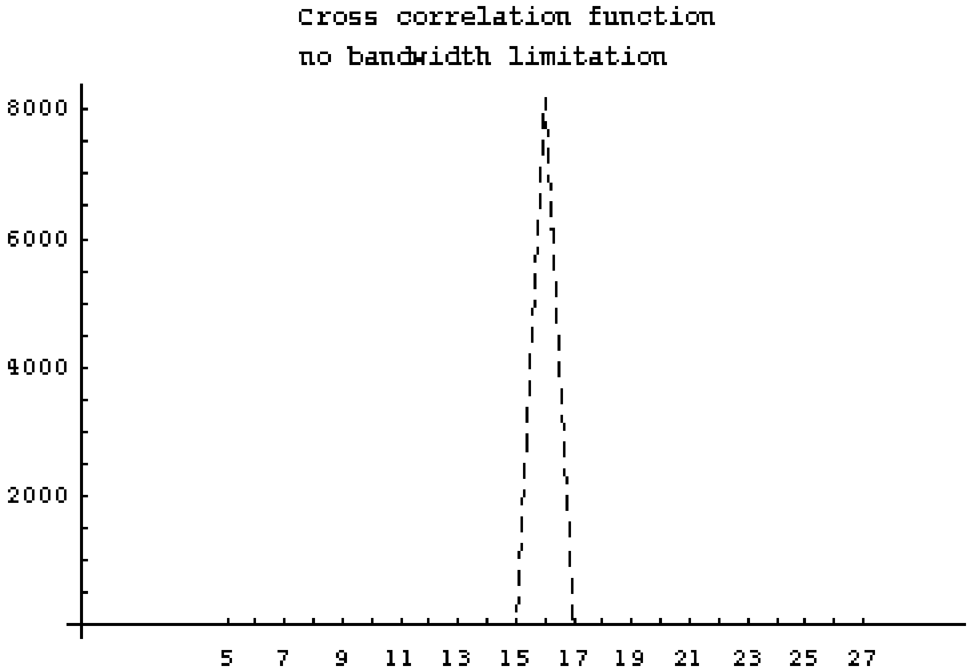

Figure 2. System receiver relative output vs target range delay. The system internal PN code, used for correlation, is delayed 16 code bits. The internal code delay determines the position of the range bin in distance from the radar by the delay of the internal code used for correlation. When the transmitted radiation time of flight delay to and from the target matches the internal code

delay, the system output is at a maximum. If the two delays are mismatched by more than ±1 code bit period, the system output is zero.

Interpretation of Cross Correlation Functions – Radar Range Resolution

Figure 2 may be used to define the range resolution of the radar. If a target is positioned in the center of bin #18, there will be no response in bin #16. If the target is moved closer than the middle of bin #18, a response, albeit small in amplitude, will begin to appear in bin #16. In a practical sense, the range resolution of this type of radar is 2 range bins.

For the case at hand, 2 range bins corresponds to 44 cm, as each range bin has a base width of 44 cm. The bin width comes about by equating a differential range delay of 22 cm to a differential time delay of 1.47 ns since the two-way radiation propagation delay is 2 R/c where “R” is range, and “c” is the speed of light.

In the practical application, various objects reflect more or less power back to the radar receiver, depending on the material composition and shape of the objects. Where there are two closely spaced objects, one may encounter a small reflector just in front of a very large reflector. It is desired that the objects be fully separated in the radar data if the objects are separated by two or more range bins, even though the farther range object may reflect 10^3 (30 db) times the amount of power than the closer object. Therefor, the farther object response must be rejected by the receiver by 30 db or more when the receiver has its range bin centered on the closer object in order to fully separate the two targets.

Note that the relative amplitude scale in figure 2 is a reference, is linear, and its maximum is directly proportional to reflected target power. Also, the cross correlation curve in fugure 2 is given in voltage, not power, as that is how the receiver functions. For a target “behind” the target in bin #16 to be 30 db less in response to the bin #16 target, its reading on the curve in figure 2 would be 260, whereas the bin #16 target will have a reading of 8191.

The central point of interest may now be re-phrased as “How much of the spectrum shown in figure 1 must be retained to prevent unacceptable distortion to the curve in figure 2?”

Computation of Necessary Bandwidth

The analysis will simply examine two cases of bandwidth limitation by comparing the cross correlation results to the ideal discussed above. This is accomplished by

1. computing the Fourier transform of the code wavefrom,
2. applying different filter functions,
3. computing the inverse transform of the filtered code waveform
4. cross correlating the filtered code waveform with the internal code waveform
5. comparing the cross correlation results to the ideal case.

Case 1 – Application of a 680 Mhz Band Pass Filter

Figure 3 shows the filter response superimposed on the radiated code spectrum; the product of the two functions is taken, but not shown. The resulting filtered code is shown in the time domain in figure 4, and the resulting cross correlation function is shown in figure 5. The bandpass filter is a simple second order filter, which doesn't limit the resulting bandwidth by significant amounts. As indicated in figure 3, the first sidelobes will only be reduced by approximately 9 db.

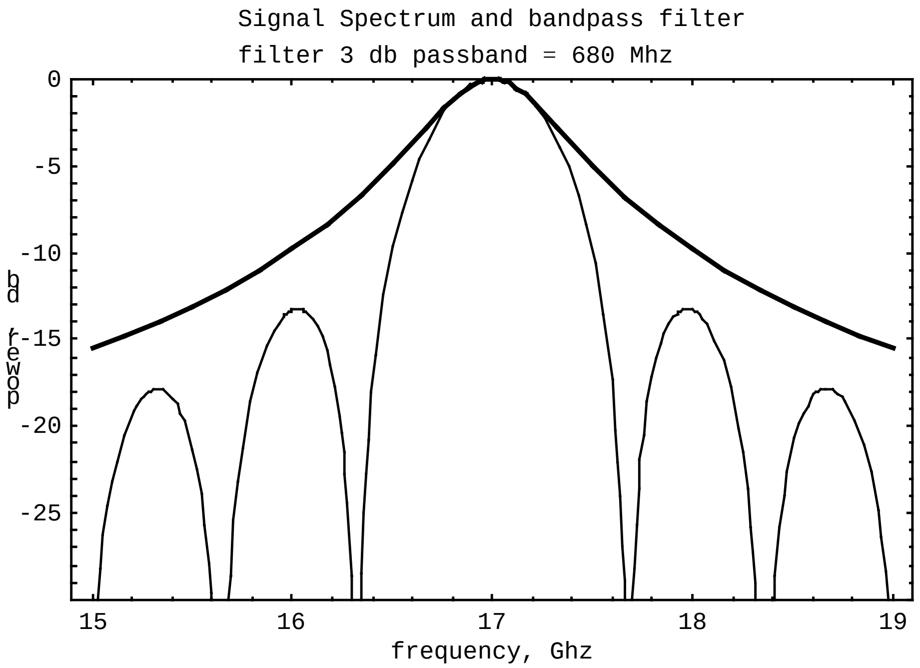

Figure 3. Signal Spectrum and a 680 Mhz bandpass filter function. The bandpass filter is applied to the signal spectrum for study.

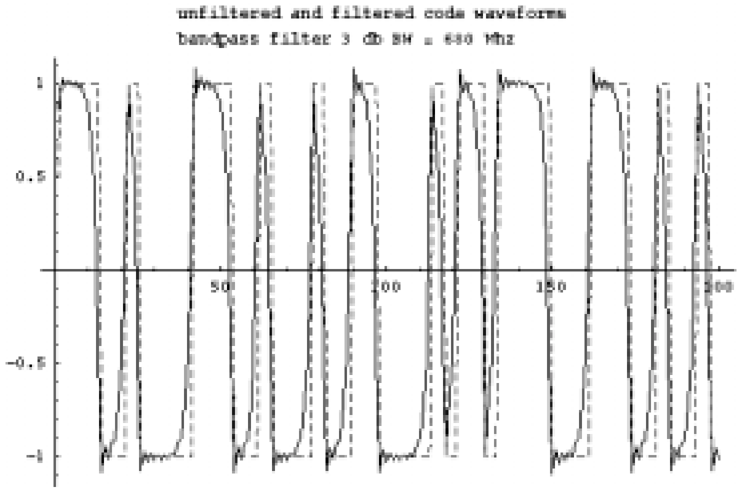

Figure 4. Comparison between the unfiltered code voltage and the bandpass filtered code voltage. Only the first 200 bits of the code are shown. The range bin response is the cross correlation between the two codes.

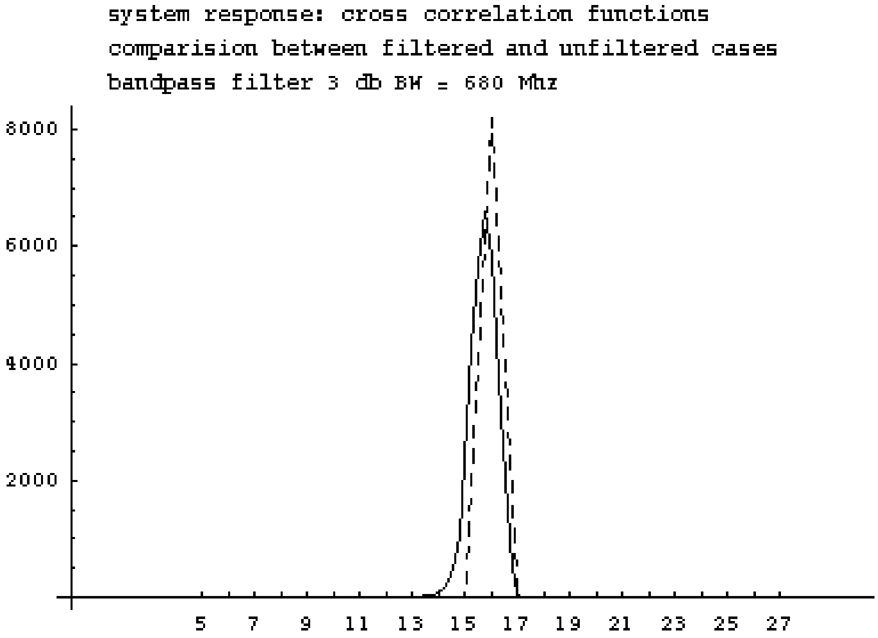

Figure 5. Resulting cross correlation function between the receiver internal code and the returned bandpass filtered code.

Figure 5 indicates unacceptable distortion of the radar range bin response. Whereas the response in bin #15 due to a target centered in bin #16 should be zero in the ideal case, it is approximately 1013. Compared to the maximum response, it is only 16.3 db down, thus target separation capability has been significantly degraded.

Case 2 – Spectral Sidelobe Elimination

In this second case, an ideal filter is applied where only the mainlobe of the spectrum shown in figure 1 is retained, i.e. only those signal frequency components of 17 Ghz ± 680 Mhz are used to construct the target return signal. Figure 6 shows the resulting time domain return code waveform, and figure 7 shows the resulting cross correlation function.

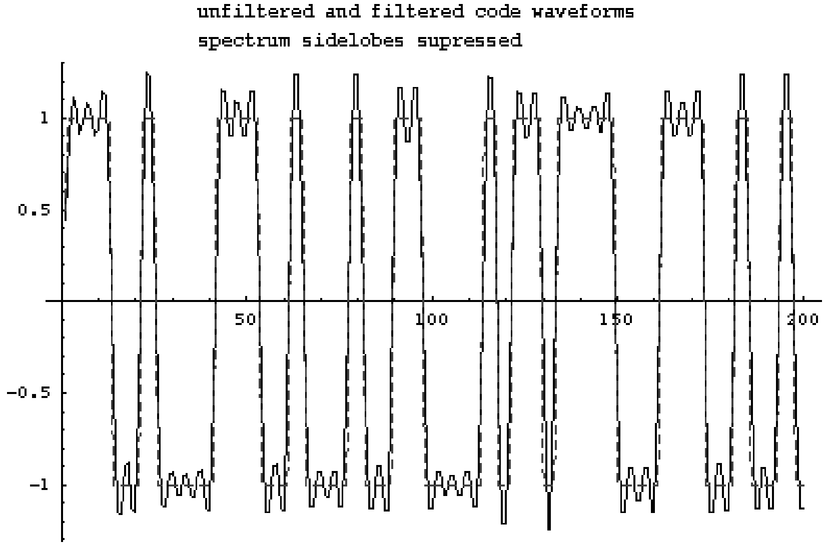

Figure 6. Filtered and unfiltered code voltages. The range bin response is the cross correlation between the two codes.

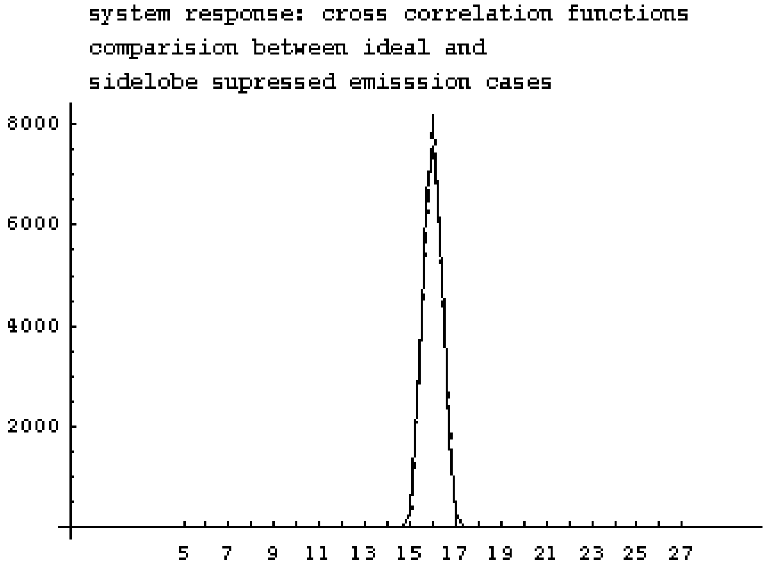

Figure 7. Correlation function comparison for the case of spectral sidelobe elimination. There is very little difference between the two range bin responses.

As can be seen in figure 7, range bin fidelity is retained even if the spectral sidelobes are eliminated entirely, so long as the spectrum mainlobe remains unfiltered. The response in bin #15 due to a target centered in bin #16 is approximately 30 db down from the system maximum desired response in bin #16. Thus this system can reject a near by target which is 1000 times larger over the target of interest in reflected power , as required.

Conclusion

A criteria for definition of necessary bandwidth in a radar employing carrier bi-phase modulation according to a serial bit stream which is pseudo random in nature has been given. The resulting conclusion per application of the criteria is that the necessary bandwidth is the mainlobe null-to-null bandwidth. The mainlobe bandwidth is twice the code bit rate frequency, hence the bit rate specifically defines the necessary bandwidth.

Figure 7 shows that the spectrum sidelobes are clearly unnecessary for proper functioning of the system, so long as the spectrum mainlobe is unattenuated. Approximately 30 db of range bin isolation is maintained by a system which radiates no spectral sidelobes whatsoever. A system with 30 db of range bin isolation is practical and will function properly in the field. This system will be able to separate closely spaced targets which vary in reflection properties by up to 30 db, which encompasses a great majority of real world cases.

With slight attenuation in the spectrum mainlobe, represented by a simple second order bandpass filter, the system range resolution degrades significantly to levels which render the system unusable . Figure 5 shows this effect.

The necessary bandwidth for a PN encoded bi-phase modulated radar waveform is no more than the spectrum mainlobe, but must include the entire mainlobe without attenuation.

References

[1] Woodward, P. M., Probability and Information Theory with Applications to Radar , Artech House, Dedham MA, 1980, Chapter 7.

[2] Taub, H. and Schilling, D.L., Principles of Communication Systems, 2nd ed., McGraw-Hill, New York, 1986, section 6.2