Exhibit G - SoFIE Topside Sounder Link Budget Analysis Results

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Executive Summary

This report documents the link budget for the Solar Flare and Ionosphere Explorer (SoFIE) 3U CubeSat topside sounder payload operating at an orbital altitude of 500 km. The radar system utilizes a center frequency of 9.303 MHz with a low-power, high-resolution linear frequency modulated (LFM) chirp to monitor ionospheric electron density changes caused by solar flares. By pairing a 1 W EIRP pulse configuration with a high-impedance active receiver architecture, the system achieves a robust link margin while operating well within the severe size, weight, and power (SWaP) constraints of a CubeSat platform.

1. System Parameters & Configuration

Spacecraft & Orbit Constraints

- **Orbital Altitude**: 500 km
- **Target Altitude (F2 Peak)**: ~300 km above the Earth
- **One-Way Distance to Target (*R*)**: 200,000 meters (500 km orbit − 300 km target altitude)
- **Wavelength (*λ*)**: 32.23 meters (@ 9.303 MHz)

Pulse Architecture

- **Center Frequency (f_c)**: 9.303 MHz
- **Chirp Bandwidth (*Δf*)**: 125.0 kHz
- **Pulse Duration (*τ*)**: 500 µs
- **Pulse Repetition Frequency (PRF)**: 7.6 Hz (Pulse Repetition Interval = 131.6 ms)
- **Transmitter Duty Cycle**: 0.38%

2. Electrically Short Nitinol Antenna Derivation

Because a 3U CubeSat platform restricts antenna mass and mechanical volume, the sounder relies on a deployable 5-meter tip-to-tip dipole fabricated from 1 mm diameter Nitinol (nickel-titanium shape-memory alloy) wire.

2.1 Directivity of an Electrically Short Dipole

At 9.303 MHz, the free-space wavelength is $\lambda = c / f = 32.23$ meters. The physical length of the antenna ($L = 5$ m) yields an electrical length of:
$L / \lambda = 5 \text{ m} / 32.23 \text{ m} = 0.155\lambda$

Any dipole below 0.5λ is governed by an electrically short current distribution, which establishes a mathematically fixed maximum directivity (D) of 1.5:
$D\_dBi = 10 \log_{10}(1.5) = 1.76$ dBi

2.2 Radiation Resistance (R_r)

For an electrically short dipole with a triangular current distribution, the radiation resistance represents the actual power converted into electromagnetic waves:
$R_r = 20\pi^2 (L / \lambda)^2 = 20 \times \pi^2 \times (0.155)^2 \approx 4.75\ \Omega$

2.3 Nitinol Ohmic Skin-Effect Loss Resistance (R_{loss})

Nitinol possesses an electrical conductivity of $\sigma \approx 1.1 \times 10^6$ S/m (50 times less conductive than pure copper). High-frequency current is forced entirely to the outer boundary of the 1 mm wire.

1. **Skin Depth (δ) Calculation**:
   $\delta = 1 / \sqrt{}(\pi \times f \times \mu\_o \times \sigma) = 1 / \sqrt{}(\pi \times (9.303 \times 10^6) \times (4\pi \times 10^{-7}) \times (1.1 \times 10^6)) \approx 157\ \mu\text{m}$
2. **RF Loss Resistance (R_{loss}) Calculation**: Using a wire radius of $a = 0.5$ mm and total dipole element length of $L = 5$ m:
   $R_{loss} = L / (2\pi\ a \times \delta \times \sigma) = 5 / (2 \times \pi \times (0.0005) \times (157 \times 10^{-6}) \times (1.1 \times 10^6)) \approx 2.80\ \Omega$

2.4 Antenna Efficiency (η) and Gain (G_{ant})

The radiation efficiency scales the absolute directivity down into actual net gain based on internal thermal heat dissipation:

- $\eta = R_r / (R_r + R_{loss}) = 4.75 / (4.75 + 2.80) = 0.629 \rightarrow -2.01$ dB
- $G_{ant} = D_{dBi} + \eta_{dB} = 1.76 \text{ dBi} - 2.01 \text{ dB} = -0.25$ dBi
  (A conservative baseline value of -0.30 dBi is applied inside the master link table).

2.5 Active Antenna High-Z Interface Factor

The antenna exhibits a massive capacitive reactance (X_a) calculated via standard short-dipole transmission line approximations:
$X_a \approx -j\ 120 / (\pi \times (L / \lambda)) \times [\ln(L / a) - 1] \approx -j\ 6{,}350\ \Omega$

By avoiding a hardware tuning inductor on receive, the 0 dB power-matching loss rule applies. However, the dipole's free-space capacitance ($C_a \approx 2.7$ pF) interacts directly with the high-impedance LNA's parasitic input capacitance ($Ci_n \approx 0.8$ pF). This establishes an un-tunable capacitive voltage divider attenuation factor:
Voltage Factor = $20 \log_{10}(C_a / (C + C_{in})) = 20 \log_{10}(2.7 / (2.7 + 0.8)) = -2.20$ dB

3. Receiver Chain & Noise Floor Analysis

The receiver utilizes a high-gain LNA to mask the noise floor of the downstream Software Defined Radio (SDR).

- **LNA Noise Figure (NF_{LNA})**: 3.0 dB ($F = 1.995$, $G = 20$ dB)
- **SDR Noise Figure (NF_{SDR})**: 15.0 dB ($F = 31.623$)
- **System Noise Figure (NF_{sys})**: 3.62 dB (*F_sys* = 2.301)
- **Receiver Noise Temperature (T_{rec})**: 377.4 K

Environmental Noise Dominance

At 9.303 MHz, the receiver is entirely external noise limited due to massive cosmic, galactic, and solar background noise.

- **Antenna Background Temperature (T_{ant})**: 500,000 K
- **Total System Temperature (T_{sys})**: ≈ 500,377 K *(Receiver hardware noise is negligible)*
- **System Figure of Merit (G / T)**: **-57.29 dB/K** *(Accounting for real-world space environment and -0.30 dBi structural antenna gain)*

4. Signal Processing & Resolution Gain

Digital Pulse Compression

Upon receiving the echo, the SDR applies a digital matched filter to compress the 500 µs chirp, yielding a significant processing gain via the Time-Bandwidth Product (TBP):
Processing Gain = $10 \log_{10}(B \times \tau) = 10 \log_{10}(125{,}000 \times 0.0005) = +17.96$ dB

Target Vertical Spatial Resolution

$\Delta R = c / (2 \times B) = (3 \times 10^8 \text{ m/s}) / (2 \times 125{,}000 \text{ Hz}) = 1{,}200$ meters

5. Master Link Budget Table

The following table contains the full mathematical model for two ionosphere reflectance scenarios at 1 W EIRP (0 dBW).

Link Budget Parameter	Nominal Value	Degraded Value (50% Reflect.)	Unit	Description / Source
Peak Radiated Power (EIRP)	0.00	0.00	dBW	Design Baseline (1 Watt)
Geometric Path Loss ($1/R^2$)	-103.86	-103.86	dB	Specular sheet reflection over 2R = 400 km
Ionospheric Reflection Efficiency (ρ)	-1.00	-3.00	dB	Plasma gradient turning point loss
Faraday Polarization Mismatch	-3.00	-3.00	dB	Elliptical rotation vs. single linear dipole
Plasma Group Retardation Penalty	-2.00	-2.00	dB	Pulse dispersion/filter mismatch loss
Antenna Structural Gain (Nitinol Short Dipole)	-0.30	-0.30	dBi	1.76 dBi Directivity minus 2.0 dB skin-effect loss
Active Antenna High-Z Interface Factor	-2.20	-2.20	dB	Capacitive voltage division factor at LNA input
Total Echo Power at LNA Input	-112.36	-114.36	dBW	Raw signal voltage power arriving at the SDR
Thermal Noise Density ($k * T_{sys}$)	-171.60	-171.60	dBW/Hz	At T_{sys} = 500,377 K (Cosmic Noise dominated)
Receiver Bandwidth (B)	50.97	50.97	dB-Hz	125.0 kHz IF window
Total Noise Power (N)	-120.63	-120.63	dBW	In-band cosmic noise floor
Pre-demodulation SNR	8.27	6.27	dB	Echo power relative to raw noise floor
Digital Pulse Compression Gain	17.96	17.96	dB	Processing gain from chirped waveform ($B * \tau$)
Final Compressed SNR	26.23	24.23	dB	Output signal after matched filtering
Required SNR for Detection	13.00	13.00	dB	Required threshold for clear identification
Net Link Margin	**13.23**	**11.23**	**dB**	**System margin**

Summary

The analysis provided demonstrates that a positive link margin is maintained for the SoFIE topside sounder after accounting for galactic background noise, ionospheric reflection losses, electrically short antenna losses, receiver noise contributions, and the benefits of matched filter compression gain. A healthy margin of 11 dB ensures additional, unaccountable losses will not affect the detection of the return pulse from the topside sounder.