Mast Height Sizing for Rooftop Corner-Reflector Installations

Ground-to-Roof Geometry with Parapet Clearance

RADLAB Technical Note

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Abstract

When a radar antenna on the ground illuminates a corner reflector mounted on a mast above a building roofline, the near parapet of the building is the binding scattering obstacle. This note derives the required mast height h_t to place the parapet at or below the first null of the antenna pattern, proves the formula with a numerical example, and catalogues the key sensitivities. The result is a single closed-form expression that accounts for the look-angle penalty inherent in steep upward illumination.

1 Problem Statement

A corner reflector (or other calibration target) is mounted atop an absorber-lined mast that extends above the roofline of a building. A radar antenna sits on the ground at horizontal range R from the near edge (parapet) of the roof, at height h_a above the ground plane. The roofline stands at height H above the antenna phase center.

The goal is to choose the mast height h_t (reflector center above the roofline) such that the near parapet falls at least outside the half-power beamwidth (HPBW) and ideally outside the null-to-null beamwidth (NNBW) of the antenna, so that the roofline does not scatter energy back into the measurement.

Because the reflector sits above the roofline and everything behind and above it is open sky, the parapet is the only geometric obstacle that constrains the mast height.

2 Geometry

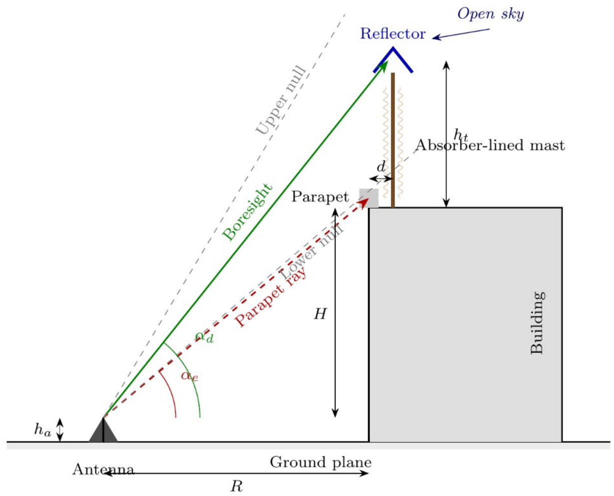

Figure 1: *Ground-to-roof geometry. The antenna on the ground looks up at the reflector on a mast above the roofline. The near parapet at elevation α_e is the binding obstacle; everything behind and above the reflector is open sky. The angular separation Δθ_e = α_d − α_e must exceed θ_NN/2.*

2.1 Definitions

With the origin at the antenna phase center and all heights measured from the ground plane:

$$\alpha_e = \arctan\frac{H}{R} \qquad \text{(elevation of the near parapet)}$$

$$\alpha_d = \arctan\frac{H+h_t}{R+d} \qquad \text{(elevation of the reflector)}$$

$$\Delta\theta_e = \alpha_d - \alpha_e \qquad \text{(angular separation, parapet to boresight)}$$

where d is the horizontal setback of the mast behind the parapet (nominally zero; positive means the mast is further from the antenna).

2.2 The Requirement

The parapet must subtend an angle from boresight that places it outside the first null:

$$\boxed{\Delta\theta_e \;\equiv\; \alpha_d - \alpha_e \;\geq\; \frac{\theta_{\mathrm{NN}}}{2}}$$

For a floor requirement (outside the HPBW only), replace θ_NN with θ_3dB.

3 Derivation of Mast Height

Set δ ≡ θ_NN/2 and take d = 0 (mast at the parapet). The equality condition of (4) gives α_d = α_e + δ, so from (2):

$$\frac{H+h_t}{R} = \tan(\alpha_e+\delta)$$

Solving for h_t and substituting H = R tan α_e:

$$h_t = R\big[\tan(\alpha_e+\delta) - \tan\alpha_e\big]$$

3.1 Closed-Form via Tangent Difference Identity

Using the identity tan A − tan B = sin(A−B) / (cos A cos B) with A = $\alpha_e + \delta$ and B = α_e:

$$\boxed{h_t = \frac{R\,\sin\delta}{\cos\alpha_e\,\cos(\alpha_e+\delta)}}$$

3.2 Small-Angle Approximation

For $\delta \ll 1$ (in radians), $\sin\delta \to \delta$ and $\cos(\alpha_e + \delta) \to \cos\alpha_e$:

$$h_t \;\approx\; \frac{R\,\delta}{\cos^2\alpha_e} \;=\; R\,\delta\,\sec^2\!\alpha_e \;=\; \frac{R_s\,\delta}{\cos\alpha_e}$$

where $R_s = R / \cos\alpha_e$ is the slant range to the parapet.

Physical interpretation. The reflector must sit a perpendicular distance $R_s\,\delta$ off the parapet ray. A vertical mast delivers only $h_t \cos\alpha_e$ of perpendicular displacement — hence the $\sec\alpha_e$ penalty on top of the slant-range stretch. Steep look angles are expensive.

Approximation error. Expanding $\cos(\alpha_e + \delta)$ shows that (8) always runs low, by a factor of approximately $(1 + \delta\tan\alpha_e)$.

4 Null-to-Null Beamwidth Sizing

For a uniformly illuminated aperture of width D, the first null occurs at $\sin\theta_1 = \lambda/D$, and $\theta_{3dB} = 0.886\lambda/D$, giving $\theta_{NN} = 2.26\,\theta_{3dB}$. With realistic illumination taper (−20 to −25 dB), the null pushes to

$$\theta_{\mathrm{NN}} \;\approx\; 2.4\,\theta_{3\mathrm{dB}}$$

Use 2.4 as the planning number and verify against a measured pattern cut.

5 Numerical Verification

5.1 Parameters

Parameter	Symbol	Value
Horizontal range	R	60 m
Roofline height above antenna	H	20 m
Half-power beamwidth	θ_{3dB}	4°
Null-to-null beamwidth	θ_{NN}	9.6°
Half null-to-null	δ	4.8°
Mast setback	d	0

5.2 Step 1: Parapet Elevation

$$\alpha_e = \arctan\frac{20}{60} = \arctan(0.33333) = 18.435^\circ$$

5.3 Step 2: Mast Height from the Tangent-Difference Form (6)

$$h_t = 60\big[\tan(23.235°) - \tan(18.435°)\big] = 60\big[0.42946 - 0.33333\big] = \mathbf{5.768}\ \mathrm{m}$$

5.4 Step 3: Verification via Closed Form (7)

$$h_t = \frac{60\ \sin(4.8°)}{\cos(18.435°)\ \cos(23.235°)} = \frac{5.02068}{0.871847} = \mathbf{5.759}\ \mathrm{m}\ \checkmark$$

The 0.009 m discrepancy is rounding in intermediate values.

5.5 Step 4: Forward Check

The reflector sits at H + h_t = 25.768 m above the antenna at R = 60 m:

$$\alpha_d = \arctan\left(\frac{25.768}{60}\right) = 23.235°, \qquad \Delta\theta_e = 23.235° - 18.435° = 4.800° = \delta\ \checkmark$$

5.6 Step 5: Independent Check via Perpendicular Offset

The parapet ray has slant length R_s = 60 / cos(18.435°) = 63.246 m. Adding a vertical mast h_t decomposes into h_t cos α_e perpendicular to the ray and h_t sin α_e along it:

$$\Delta\theta_e = \arctan\frac{5.768 \times 0.94868}{63.246 + 5.768 \times 0.31623} = \arctan\frac{5.4718}{65.070} = 4.81°\ \checkmark$$

5.7 Step 6: Approximation Accuracy

$$h_t^{\mathrm{approx}} = R\,\delta\ \sec^2\alpha_e = 60 \times 0.08378 \times 1.1111 = 5.585\ \mathrm{m}$$

This is 3.2% low, consistent with the predicted correction factor (1 + δ tan α_e) = 1 + 0.08378 × 0.33333 = 1.028.

6 Sensitivity Analysis

6.1 Setback is a Cost, Not a Mitigation

With d ≠ 0, requiring α_d = α_e + δ gives H + h_t = (R + d) tan(α_e + δ), so:

$$\frac{\partial h_t}{\partial d} = \tan(\alpha_e + \delta) = 0.4295\ \mathrm{m/m}$$

Every metre the mast is set back behind the parapet costs 0.43 m of additional mast height.

Verification. At d = 3 m and h_t = 5.768 m: α_d = arctan(25.768/63) = 22.245°, so Δθ_e = 3.81° — inside the null. Restoring Δθ_e = 4.8° requires h_t = 63 × 0.42946 − 20 = 7.06 m, an increase of +1.29 m for +3 m of setback. This is the opposite of the rooftop-to-rooftop case, where setback helped. Here: cantilever the reflector out over the parapet if possible.

6.2 Look Angle Dominates

Since h_t ∝ sec² α_e in the small-angle regime, reducing R from 60 m to 30 m raises the look angle from 18.4° to 33.7° (sec² increases by 30%). The result:

$$h_t\big|_{R=30} = 30[\tan(38.49°) - \tan(33.69°)] = 30[0.7955 - 0.6667] = 3.86\ \mathrm{m}$$

A net win (halving R reduces h_t by 33%, not 50%), but less than the naive linear-in-R scaling suggests. Moving the antenna closer also risks violating the far-field condition $R \geq 2D^2/\lambda$.

6.3 Ground Bounce is Geometrically Free

Image the reflector at depth −(2h_a + H + h_t) below the antenna; the bounce ray departs at depression angle arctan((2h_a + H + h_t)/R), giving:

$$\Delta\theta_g = \alpha_d + \arctan\frac{2h_a + H + h_t}{R} \approx 2\alpha_d \approx 46.5^\circ$$

This is roughly 4.8 θ_NN — deep in the far sidelobe region. The ground bounce cannot be corrected by mast height and does not need to be; treat it as a sidelobe-level and absorber-mat problem, not a geometry problem.

7 Design Table

Mast height h_t (exact, from (7)) for various beamwidths and ranges, with H = 20 m:

θ_3dB (°)	θNN (°)	h_t at R=30 m	h_t at R=60 m	h_t at R=100 m
2	4.8	1.91	2.85	3.77
4	9.6	3.86	5.77	7.63
6	14.4	5.87	8.78	11.63
7	16.8	7.10	10.35	13.53
10	24.0	10.10	15.20	20.32
20	48.0	23.18	36.02	49.85

Table 1: *Required mast height above roofline for null-to-null clearance of the near parapet. Building height H = 20 m above the antenna phase center.*

8 Signal-to-Clutter Considerations

Placing the parapet at the first null is a geometric condition, not an SCR guarantee. For monostatic measurement at essentially equal range:

$$\mathrm{SCR} = \frac{\sigma_{\text{ref}}}{\sigma_{\text{edge}}} \cdot \frac{1}{g^4(\Delta\theta_e)}$$

where g is the normalized voltage pattern. Note the fourth-power dependence — one-way sidelobe suppression buys double in dB of two-way rejection.

A vertical parapet face of illuminated area A viewed near-normal has plate-like RCS $\sigma = 4\pi A^2/\lambda^2$. At X-band, even a modest 1 m² illuminated patch gives σ_edge ~ +50 dBsm. Against a 30 cm trihedral (σ_ref ≈ +10 dBsm), achieving 20 dB SCR requires ~60 dB of two-way suppression, or −30 dB one-way.

The null is a few tenths of a degree wide in practice — radome effects, mast scattering, and finite Fresnel-zone extent on the parapet all smear it. **Size h_t to the null, but verify against the measured near-in sidelobe envelope.** Assume you land on the first sidelobe (−13 dB uniform, −25 to −30 dB tapered), not in the null notch.

9 Practical Mitigations

When the required mast height is impractical, ranked by leverage:

1. Cantilever the reflector out past the parapet. Negative d gives free angular separation. Each metre of overhang saves ~0.43 m of mast.

2. Squint the beam up. Point boresight 0.25 θ_3dB above the reflector: ~1 dB one-way loss on the target, but 8–15 dB two-way gain against the parapet (steep part of the pattern skirt).

3. Rolled-edge or absorber-draped parapet. A rolled edge (radius ≳ several λ) converts a plate/edge specular into a smoothly-varying creeping-wave return. Far more effective than flat absorber on a sharp corner.

4. Coherent background subtraction. Measure with the reflector removed (or rotated to a low-RCS aspect); vector-subtract. Kills the static edge return regardless of pattern. Does not kill multipath ripple.

5. Time-domain gating. Via stepped-frequency + IFFT, softly windowed. With bandwidth B, resolves ΔR = c/2B; at B = 1 GHz, ΔR = 15 cm.

10 Summary

For a ground-based antenna looking up at a rooftop reflector, the mast height above the roofline required to clear the near parapet by one half of the null-to-null beamwidth is:

$$\boxed{h_t = \frac{R\,\sin(\theta_{\rm NN}/2)}{\cos\alpha_e\,\cos(\alpha_e + \theta_{\rm NN}/2)}}$$

with approximation $h_t \approx R\,(\theta_{NN}/2)\sec^2\alpha_e$, accurate to ~3% for $\theta_{NN} \le 10°$ and $\alpha_e \le 35°$.

Key takeaways:

• Setback hurts — putting the mast further behind the parapet increases the required height at ~0.43 m per metre of setback (at the example geometry).

• The $\sec^2\alpha_e$ factor makes steep look angles expensive; moving the antenna closer saves less mast than the naive linear scaling suggests.

• The ground bounce is geometrically self-clearing (tens of beamwidths off boresight) and is an absorber problem, not a mast-height problem.

• Size h_t to the null, but verify against the measured sidelobe envelope — the null is too narrow and fragile to depend on operationally.

11 Attachments

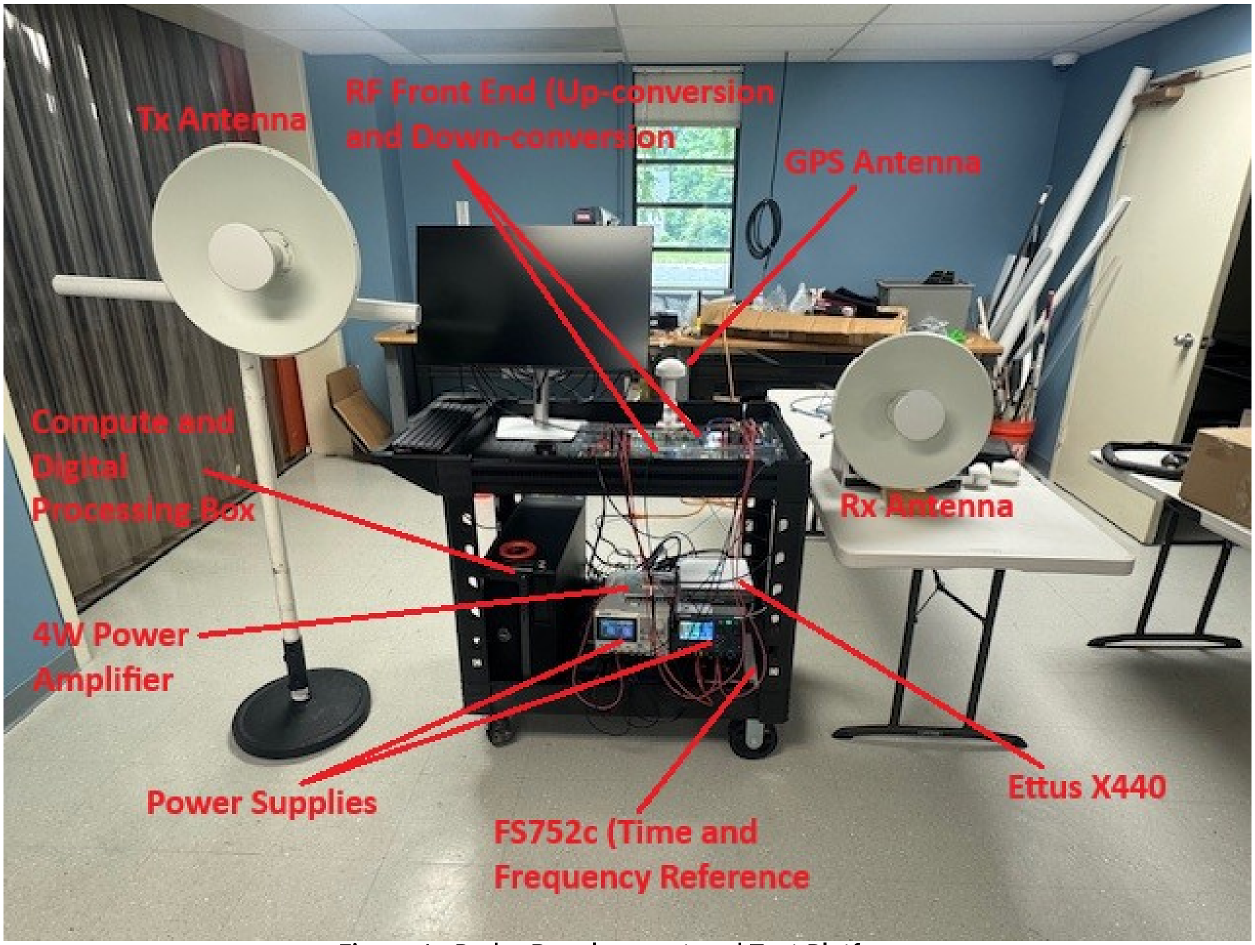

Figure 1. Radar Development and Test Platform.

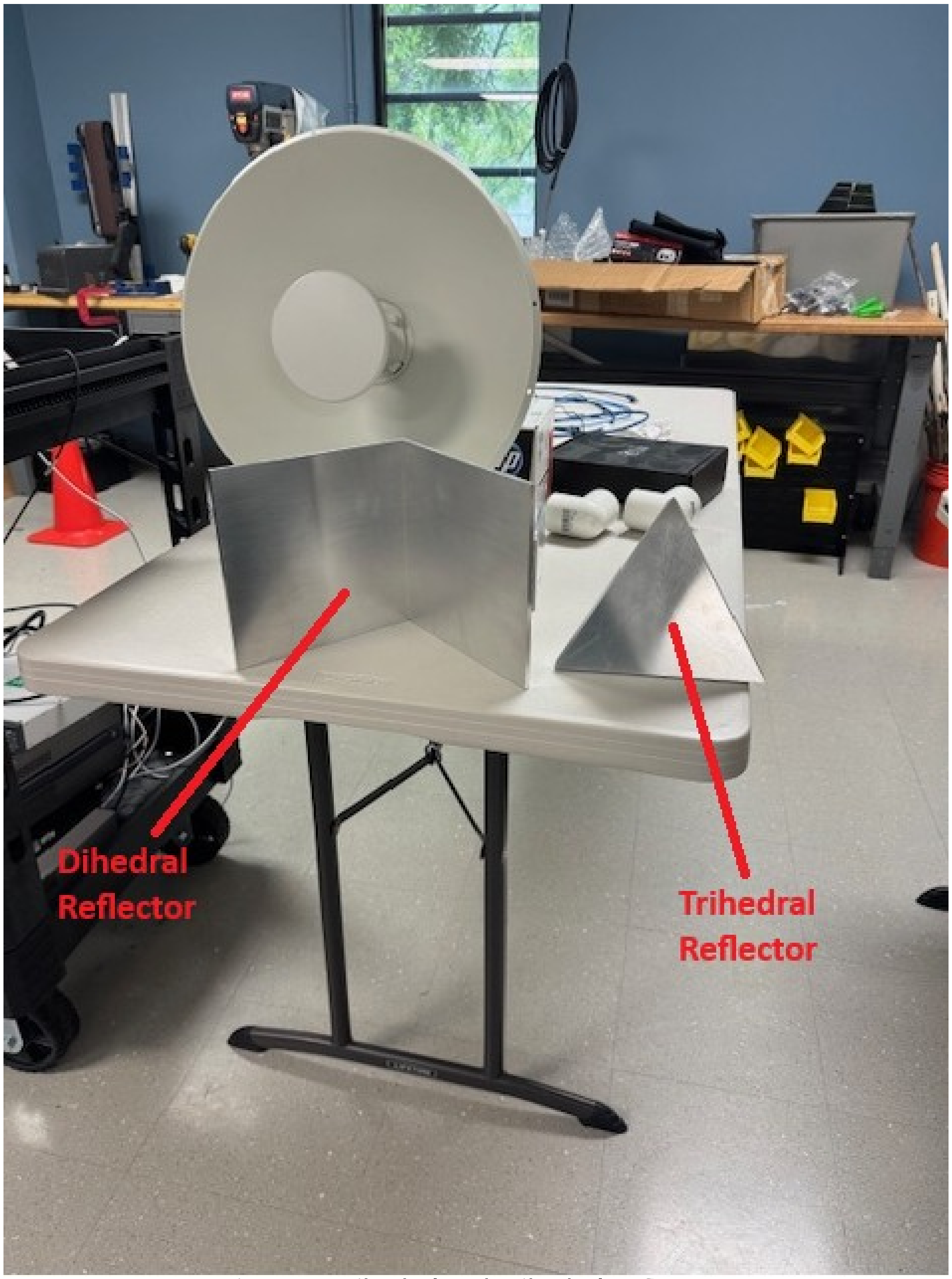

Figure 2. Dihedral and Trihedral Reflectors