FCC Environmental Assessment: Ignis Mission

Technical Supplement

ELS File Number 0330-EX-CN-2019

November 5, 2019

Introduction

Astro Digital US, Inc. ("Astro Digital") hereby supplements its revised environmental analysis ("Revised EA"), submitted to the Federal Communications Commission ("FCC") on September 30, 2019 in the above-referenced proceeding. The following analysis addresses the conclusion in the Revised EA (at p. 9) that matter in the atmosphere will not have a significant impact on the trajectory of the mercury ions ejected from the spacecraft. Consideration has been given to energy loss of mercury ions as they travel through the thin atmosphere in low-Earth orbit as well as trajectories based on when the thruster is fired in a given orbital period.

Detailed Analysis

In order to calculate the distance the mercury ion travels until its velocity is reduced to below Earth escape velocity, we model a mercury ion passing through a gaseous medium (which approximates the atmosphere in low-Earth orbit). The following table lists the key assumptions used for this analysis:

Table 1. Key Assumptions and Parameters

Parameter	**Value**	**Source**
Target medium	100% atomic oxygen	Introduction to Atmospheric Chemistry, Hobbs, p. 29[1]
Altitude	500 km	Nominal launch altitude
Target medium density (low solar activity)[2] at 500 km altitude	6.0E-17 g/cm^3	Robert A. Braeunig[3]
Ion	Hg	ACE thruster propellant
Ion energy	300 eV	Parameter provided by ACE thruster
Ion charge (q)	1.6E-19 Coulombs	Physical property of propellant
Ion mass (m)	3.33E-25 kg	Physical property of propellant
Stopping power	Nuclear stopping power dominates at 300 eV	

We use the SRIM ("Stopping and Range of Ions in Matter") software to model the trajectory of the mercury ion in the atmosphere at 500 km orbital altitude. The SRIM software was introduced over 30 years ago and is widely used for this type of analysis.[4]

[1] Peter Victor Hobbs, Introduction to Atmospheric Chemistry (Cambridge University Press 2000)

[2] The NOAA Space weather prediction center data shows that we have been in a low solar activity mode for the past few years. https://www.swpc.noaa.gov/phenomena/f107-cm-radio-emissions.

[3] http://www.braeunig.us/space/atmos.htm.

According to “Introduction to Atmospheric Chemistry” by Hobbs (p. 29), the atmosphere at 500 km altitude is predominantly atomic oxygen (AMU of 16). Accordingly, we have modeled the mercury ion traveling through atomic oxygen.

We used 300 eV for the ion energy associated with our 300 V Hall thruster (*i.e.*, the positive charged ion atoms are accelerated by a 300V potential provided by the ACE thruster).

The equation relating the charge, voltage, and velocity of the mercury ion is as follows (NASA Goddard[5]):

$$\mathbf{qE = (½)\ m\ V^2} \qquad (\textbf{Equation 1})$$

where:

q = charge;
E = Voltage
m = ion mass; and
V = Ion exit velocity from thruster.

The mercury ion’s total velocity is the sum of the Ion’s exit velocity (17.0 km/s) and the Ignis satellite’s orbital velocity (7.6 km/s) or 24.6 km/s (= 17.0 km/s + 7.6 km/s).

Using **Equation 1**, the total energy of the mercury ion is **630 eV**. Accordingly, the mercury ion’s initial conditions are:

Velocity = 24.6 km/s
Energy = 630 eV

The End Condition (in SRIM) of the mercury ion is set to 11 km/s (equal to the Earth’s escape velocity at 500 km altitude). The mercury ion’s energy with an 11 km/s velocity is **126 eV** (using Equation 1).

The total energy loss required to decelerate the mercury ion to the Earth’s escape velocity (based on an altitude of 500 km) is **504 eV** (= 630 eV – 126 eV).

Under the key assumptions and parameters specified in Table 1 above, for a mercury ion with total energy (exit velocity + orbital velocity) of 630 eV, the nuclear stopping power is 1.12 MeV mg/cm^2 or 6.71*10^{-3} eV/km as calculated by the SRIM model.

The mean distance for the ion to decelerate to Earth’s escape velocity is calculated by dividing the change in energy required to decelerate the mercury ion to the Earth’s

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4 https://www.sciencedirect.com/science/article/pii/S0168583X10001862. According to Science Direct, “Currently, more than 700 scientific citations are made to SRIM every year.” *Id*.

5 https://www.nasa.gov/sites/default/files/files/SMI_Problem20.pdf.

escape velocity (504 eV) by the nuclear stopping power (6.71*10^{-3} eV/km) or **75,070 km**. This means that a mercury ion must travel a mean distance of 75,070 km before it decelerates to 11 m/s (Earth's escape velocity at 500 km altitude). If the mercury ions travel tangent to the orbit, they will reach an orbital altitude of 69,012 km before they are decelerated to 11 m/s (based on the calculated mean distance of 75,070 km).

The analysis above is conservative. First, the analysis uses the Earth's escape velocity at a 500 km altitude, which is significantly higher than escape velocity at a 69,012 km altitude. Second, the analysis assumes a constant atmospheric density of 6.0*10^{17} g/cm^3, which is derived from a 500 km altitude. The actual atmospheric density decreases exponentially with increasing altitude. Accordingly, on average the ions will be at an altitude of 69,012 km and traveling away from the earth by the time they have decelerated to escape velocity.

A further analysis was conducted to determine the effect of firing the thruster at different portions of the orbit (*i.e.*, during the orbit arc before and after apogee). The ACE thruster will burn for 90 seconds each burn during this mission. Since the orbital period is approximately 90 minutes (at 500 km altitude), each burn will take 1/60th of the orbit (90 seconds/90 minutes) or 6 degrees out of the 360 degree orbit. For nominal operations, the burn will start about 3 degrees before apogee is reached and last until about 3 degrees after apogee is passed. However, the mercury ions will still be traveling away from the Earth in a direction tangential to the orbital path. The effect of firing slightly before or after apogee will increase the velocity of the mercury ions because the orbital velocity is determined by the following equation (source: https://en.wikipedia.org/wiki/Elliptic_orbit):

Velocity [edit]

Under standard assumptions the orbital speed (v) of a body traveling along an **elliptic orbit** can be computed from the vis-viva equation as:

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

where:

- μ is the standard gravitational parameter,
- r is the distance between the orbiting bodies.
- a is the length of the semi-major axis.

As the satellite moves away from apogee, the r in the above equation will decrease and the orbital velocity will increase. And because the velocity of the mercury ions is the sum of the exit velocity of the ions from the thrusters plus the orbital velocity, the velocity of the mercury ions traveling away from the Earth will increase during the burn occurring both before and after apogee.